Practice problems for introduction to matrix algebra with partial answers

1. Calculate the inverse of the 22 matrices below using solve(). Make sure you can also perform the calculations using the by-hand formula given in the notes.
   1. 

> A <- matrix(data = c(1, 0.5, 0.5, 1.25), nrow = 2, ncol = 2, byrow = TRUE)

> A

[,1] [,2]

[1,] 1.0 0.50

[2,] 0.5 1.25

> solve(A)

[,1] [,2]

[1,] 1.25 -0.5

[2,] -0.50 1.0

* 1. 
  2. 

1. Why can’t inverses be found of the following matrices?
   1. 

Let the first column be denoted by **A**1 and the second column be denoted by **A**2. Notice that **A**1 = 0.5\***A**2.

* 1. 

Need a square matrix!

1. For each matrix in 1), also complete the following where you make sure you understand how to do by-hand calculations in addition to those performed in R:
   1. Verify that the matrix times its inverse results in the identity matrix
   2. Find the eigenvalues
   3. Find the eigenvectors (just using R) and verify they satisfy their corresponding equality
   4. Find the length of the eigenvectors
   5. Show that the eigenvectors are perpendicular to each other (use a plot or show that the product of the vectors is 0)
   6. Show the determinant of the matrix is equal to the product of the eigenvalues
   7. Show the trace of the matrix is equal to the sum of the eigenvalues
   8. Determine if the matrix is positive definite
2. This problem continues the diamond data set problem from the previous homework set. These parts come from a project that I gave to my STAT 870 students.

Consider the following regression model: E(Price) = β0+β1Carat. Using matrix methods (without the lm() function), find

* 1. 

 = (**X**′**X**)-1**X**′**Y** = (-1316.73, 6645.02)′

> #Read in the data

> diamonds <- read.csv(file = "c:\\data\\DiamondPrices.csv")

> head(diamonds)

carat color clarity certify price

1 0.30 D VS2 GIA 745.9184

2 0.30 E VS1 GIA 865.0820

3 0.30 G VVS1 GIA 865.0820

4 0.30 G VS1 GIA 721.8565

5 0.31 D VS1 GIA 940.1322

6 0.31 E VS1 GIA 890.8626

> X <- cbind(1, diamonds$carat)

> Y < -diamonds$price

> beta.hat <- solve(t(X)%\*%X) %\*% t(X)%\*%Y

> beta.hat

[,1]

[1,] -1316.734

[2,] 6645.024

* 1. Estimated price for carat = 0.5.

> X.h <- c(1, 0.5) #Vector containing the carat = 0.5 value

> Y.hat <- X.h%\*%beta.hat

> Y.hat

[,1]

[1,] 2005.778

