Equations for STAT 878

1. $\gamma(s,t) = Cov(x_s, x_t)$ or $\gamma(h) = Cov(x_t, x_{t+h})$
2. $\rho(s,t) = \frac{\gamma(s,t)}{\sqrt{\gamma(s,s)}\sqrt{\gamma(t,t)}}$ or $\rho(h) = \frac{\gamma(h)}{\gamma(0)}$
3. For $x_t = \mu + \sum_{j=-\infty}^{\infty} \psi_j w_{t-j}$ with $\sum_{j=-\infty}^{\infty} |\psi_j| < \infty$ and w_t ~ independent $N(0, \sigma_w^2)$, $\gamma(h) = \sigma_w^2 \sum_{j=-\infty}^{\infty} \psi_{j+h}\psi_j$ for $h \geq 0$ and $\gamma(h) = \gamma(-h)$
4. $\hat{\gamma}(h) = \frac{1}{n}\sum_{t=1}^{n-h}(x_{t+h} - \bar{x})(x_t - \bar{x})$
5. $\hat{\rho}(h) = \hat{\gamma}(h)/\hat{\gamma}(0)$ has an approximate normal distribution with mean 0 and standard deviation $\sigma_{\hat{\rho}(h)} = 1/\sqrt{n}$ if $\rho(h)=0$
6. $(1 - B)x_t = x_t - x_{t-1}$
7. $B^k x_t = x_{t-k}$
8. $\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$ for $|a|<1$
9. $(1-\varphi_1 B-\varphi_2 B^2-\ldots-\varphi_p B^p)(1-B)^d x_t = (1+\theta_1 B+\theta_2 B^2+\ldots+\theta_q B^q)w_t$ and $\varphi(B)(1-B)^d x_t = \theta(B)w_t$
10. $\alpha = \mu(1-\varphi_1-\varphi_2-\ldots-\varphi_p)$
11. Quadratic formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $ax^2 + bx + c = 0$
12. $\gamma(h) = \sum_{j=1}^{p}\varphi_j\gamma(h-j) + \sigma_w^2\sum_{j=h}^{q}\theta_j\psi_{j-h}$ for $0 \leq h < \max(p, q+1)$ and $\gamma(h)=\sum_{j=1}^{p}\varphi_j\gamma(h-j)$ for $h \geq \max(p, q+1)$
13. φ_{hh} denotes the partial autocorrelation at lag h
14. The estimated partial autocorrelation at lag h:

    $\hat{\varphi}_{11} = \hat{\rho}(1)$

    $$\hat{\varphi}_{hh} = \frac{\hat{\rho}(h) - \sum_{j=1}^{h-1}\hat{\varphi}_{h-1,j}\hat{\rho}(h-j)}{1 - \sum_{j=1}^{h-1}\hat{\varphi}_{h-1,j}\hat{\rho}(h-j)} \text{ for } h = 2, 3, \ldots$$

    where $\hat{\varphi}_{hj} = \hat{\varphi}_{h-1,j} - \hat{\varphi}_{hh}\hat{\varphi}_{h-1,h-j}$ for h=3,4,...; j=1,2,...h-1

15. $\tilde{x}_{n+m}^{n} = E(x_{n+m}|I_n)$

16. Forecast error = $x_{n+m} - \tilde{x}_{n+m}^{n}$
17. $Var\left[x_{n+m} - \tilde{x}_{n+m}^{n}\right] = \sigma_w^2\sum_{i=0}^{m-1}\psi_i^2$
18. $(1-\alpha)100\%$ C.I. for x_{n+m}: $\tilde{x}_{n+m}^{n} \pm Z_{1-\alpha/2}\sqrt{\widehat{Var}(x_{n+m} - \tilde{x}_{n+m}^{n})}$
19. Drift term: δ
20. Standardized residuals: $e_t = \left(x_t - \tilde{x}_t^n\right)\Big/\sqrt{\widehat{Var}(x_t - \tilde{x}_t^n)}$
21. Ljung-Box-Pierce hypothesis test statistic is $Q = n(n+2)\sum_{h=1}^{H}\frac{\hat{\rho}_e^2(h)}{n-h}$; under the assumption that the null hypothesis is true, Q~ χ_{H-p-q}^2 for large n
22. $\Phi(B^s)\varphi(B)(1-B^s)^D(1-B)^d x_t = \Theta(B^s)\theta(B)w_t$

23. $(1-B)^d x_t = \left(\sum_{j=0}^{\infty} \pi_j B^j\right) x_t = \sum_{j=0}^{\infty} \pi_j x_{t-j}$ where $\pi_j = \dfrac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)}$ and $\pi_0 = 1$

24. $\pi_{j+1} = \dfrac{(j-d)\pi_j}{(j+1)}$ and $\pi_0 = 1$

25. $x_t = \sum_{j=0}^{\infty} \psi_j w_{t-j}$ where $\psi_j = \dfrac{\Gamma(j+d)}{\Gamma(j+1)\Gamma(d)}$

26. $\rho_{xy}(s,t) = \dfrac{\gamma_{xy}(s,t)}{\sqrt{\gamma_x(s,s)}\sqrt{\gamma_y(t,t)}}$; if the time series is jointly stationary, then $\rho_{xy}(h) = \dfrac{\gamma_{xy}(h)}{\sqrt{\gamma_x(0)}\sqrt{\gamma_y(0)}}$

27. $\gamma_{xy}(h) = E[(x_{t+h} - \mu_x)(y_t - \mu_y)] = Cov(x_{t+h}, y_t)$

28. $\hat{\gamma}_{xy}(h) = \dfrac{1}{n}\sum_{t=1}^{n-h}(x_{t+h} - \bar{x})(y_t - \bar{y})$

29. $\hat{\rho}_{xy}(h) = \dfrac{\hat{\gamma}_{xy}(h)}{\sqrt{\hat{\gamma}_x(0)}\sqrt{\hat{\gamma}_y(0)}}$ has an approximate normal distribution with mean 0 and standard deviation of $\sigma_{\hat{\rho}_{xy}} = 1/\sqrt{n}$ if the sample size is large and at least one of the series is white noise

30. $y_t = \beta_0 + \beta_1 z_{t1} + \ldots + \beta_r z_{tr} + x_t$

31. $y_t = \sigma_t \varepsilon_t$ where $\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \ldots + \alpha_m y_{t-m}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_r \sigma_{t-r}^2$