**Estimation – Method of Moments**

Estimate the parameters (ϕ’s, θ’s, μ, and ) of an ARIMA model. A ^ is placed above parameters to denote estimated quantities.

Method of moments estimates

Sample moments (for example, the sample mean, sample variance,…) are substituted for their corresponding population counterparts to find parameter estimates.

Example: Let X1, …, Xn ~ ind. Exponential(θ); i.e.  for 0 < x < ∞.

One can show that μ = E(X) = 1/θ. This implies that θ = 1/μ. Let  denote the sample mean. Then .

In deriving the PACF, we examined an autoregressive representation of

 where wt ~ ind. (0,)

This led to



Note: These are called the Yule-Walker equations.

For a specific p, we can solve for the ϕ’s:

* Suppose p = 1, then ρ(1) = ϕ1ρ(0) ⇒ ϕ1 = ρ(1) ⇒ 
* Suppose p = 2, then



⇒ 

In addition,  where the estimate of  results from:



Note that E(xtwt) = E[(ϕ1xt-1+…+ϕpxt-p+wt)wt] = E() = .



Suppose E(xt) = μ ≠ 0. Then  and .

Example: AR(1) with ϕ1 = 0.7, μ = 0, and  = 1 (fit\_AR1.R, AR1.0.7.txt)

Data was simulated from an AR(1) model of the form xt = 0.7xt-1 + wt where wt ~ ind. N(0,1).



The data is in x.

> rho.x <- acf(x = x, type = "correlation", main =

expression(paste("Data simulated from AR(1): ", x[t] ==

0.7\*x[t-1] + w[t], " where ", w[t], "~N(0,1)")))

> rho.x

Autocorrelations of series 'x', by lag

0 1 2 3 4 5 6 7

1.000 0.674 0.401 0.169 -0.023 -0.125 -0.067 -0.064 –

8 9 10 11 12 13 14

0.058 0.005 -0.044 -0.041 -0.017 0.064 0.076

15 16 17 18 19 20

0.160 0.191 0.141 0.081 0.006 -0.132

> gamma.x <- acf(x = x, type = "covariance", main =

expression(paste("Data simulated from AR(1): ",

x[t] == 0.7\*x[t-1] + w[t], " where ", w[t], "~N(0,1)")))

> gamma.x

Autocovariances of series 'x', by lag

0 1 2 3 4 5 6 7 8 9

2.5033 1.6864 1.0036 0.4219 -0.0586 -0.3120 -0.1687 -0.1608 -0.1445 0.0133

10 11 12 13 14 15 16 17 18 19

-0.1102 -0.1029 -0.0430 0.1599 0.1892 0.3997 0.4790 0.3529 0.2015 0.0140

20

-0.3293

> mean(x)

[1] -0.4963419

Therefore,  = 2.5033,  = 0.6737, and   
 = -0.4963. From the previous derivation, note that . Thus, the estimated model can be written as

(1 - B)xt =  + wt where wt ~ ind. (0,)

(1 - 0.6737B)xt = -0.16 + wt where wt ~ ind. N(0, 1.3671)



Note: We would use  if we believed that E(xt) ≠ 0. In most applications, you will not know it is 0! I included here using .

Compare the estimates to the actual ϕ1 = 0.7, μ = 0, and = 1 used to generate the data.

Example: MA(1)

Remember that . Then this equation can be solved for θ1. Substituting estimates for the parameters produces:



from the quadratic formula which gives two different solutions. Choose the solution that satisfies the invertibility condition (i.e., -1 < θ1 < 1).

Suppose . Then an estimate cannot be found!

From earlier in the course, . Therefore,

.

Notes:

* We do not NEED to assume that wt has a normal distribution!
* Method of moment estimators usually are not used as “final” estimates for the parameters. Instead, they are often used as initial estimates for iterative parameter estimating procedures.
* Estimates for AR only models are “optimal”. Estimates for models containing MA parameters are not “optimal”. Optimal here refers to having smallest variance.
* The R function, ar(), will do the Yule-Walker estimation method for autoregressive models (see method = "yule-walker" argument). It uses Akaike’s information criterion to choose a “best” model.