**More on expected values for linear combinations of random variables**

This section discusses some items we have already discussed and also some new items. The main purpose here is for you to get comfortable with finding expectations and variances of functions of random variables.

Suppose X and Y are random variables.

1. If a and b are constant, then

E(aX+b) = aE(X) + b

This was introduced earlier. Note what happens if a and/or b is equal to 0.

1. A more general version of 1: Suppose g(X) is any linear combination of constants with X. We can continue to distribute the expectation through the linear combination.

For example, let g(X) = aX2 + bX + c - dX - f for some constants a, b, c, d, and f. Then

E[g(X)] = E(aX2 + bX + c - dX - f)

= aE(X2)+ bE(X) + E(c) - dE(X) - E(f)

= aE(X2)+ bE(X) + c - dE(X) - f

Note that you cannot distribute an expectation through a product of random variables. For example, E(X2) ≠ E(X)E(X) usually. Why?

Suppose X is a continuous random variable.



1. A more general version of 2 with two random variables: Let X and Y be two independent random variables and let g(X,Y) be a linear combination of constants with X and Y. We can continue to distribute the expectation through the linear combination!

For example, let g(X,Y) = aX2 + bY + c + dX + f + XY. Then

E(aX2 + bY + c + dX + f + XY)

= aE(X2)+ bE(Y) + E(c) + dE(X) + E(f) + E(XY)

= aE(X2)+ bE(Y) + c + aE(X) + f + E(XY)

Note that you cannot distribute an expectation through a product of random variables except under special conditions. For example, E(XY) ≠ E(X)E(Y) as previously discussed.

1. Let X and Y be two independent random variables. Then E(XY) = E(X)E(Y).

We discussed this case earlier in this section.

Notice that  Under independence, this simplifies to





 since xg(x) has no values of y in it.

Also, remember that



Notice what happens to the covariance and correlation coefficient when X and Y are independent!

Cov(X,Y) = σXY = E(XY) – E(X)E(Y) = 0 under independence!

 under independence!

1. If a and b are constants, then  = a2σ2. Stated differently, Var(aX + b) = a2Var(X).

 Proof:



1. If a and b are constants and X and Y are random variables with joint PDF of f(x,y), then



Stated differently,

Var(aX + bY) = a2Var(X) + b2Var(Y) + 2abCov(X,Y).

Proof: Go through it on your own! Follow a similar process as with the last proof. In the proof, get to a point where



and then multiply the square portion within { } out.

1. If X and Y are independent, then Var(aX+bY) = a2Var(X) + b2Var(Y) because Cov(X,Y) = σXY = 0.

This will help us later in the course when deriving the test statistic used in a hypothesis test for two means.

Example: Grades for two courses (Cov.ipnyb)

Let X be a random variable denoting grade in a math course and Y be a random variable denoting grade in a statistics course. Suppose the joint PDF is



This is not necessarily a realistic example, but find
Var(X + 2Y):

Var(X+2Y) = Var(X) + 22Var(Y) + 2×1×2×Cov(X,Y)

= 0.0819 + 4×0.0667 + 4×(-0.0139)

= 0.2931

Sage:



1. If X1, X2, …, Xn are random variables from a joint PDF of f(x1, x2, …, xn) and a1, a2, …, an are constants, then

Var(a1X1 + a2X2 + … + anXn)

=  + 

=  + 

1. If X1, X2, …, Xn are independent random variables, then Var(a1X1 + a2X2 + … + anXn) = 